INTRODUCTION TO TOPOLOGICAL K-THEORY
EXERCISE SESSION 9
June 7, 2016

Problem 1. Choose a basepoint $x_0 \in S^n$. Show that the reduced cone $C(S^n, x_0)$ is homeomorphic to the closed $(n+1)$-disk D^{n+1}.

Problem 2. Show that the reduced suspension ΣS^n is homeomorphic to S^{n+1} for all $n \geq 0$

Problem 3. Show that $X \wedge (Y \vee Z)$ is homeomorphic to $(X \wedge Y) \vee (X \wedge Z)$ when X, Y and Z are pointed compact Hausdorff spaces. To what extent is the assumption that the spaces are compact Hausdorff necessary?

Problem 4. Let X and Y be pointed compact Hausdorff spaces. Show that the inclusions $i_X \colon X \to X \vee Y$ and $i_Y \colon Y \to X \vee Y$ induce an isomorphism

$$\tilde{K}_{\mathbf{F}}(X \vee Y) \xrightarrow[\approx]{(i_X^*, i_Y^*)} \tilde{K}_{\mathbf{F}}(X) \times \tilde{K}_{\mathbf{F}}(Y).$$

Problem 5. Use reduced real K-theory to show that $S^1 \vee S^1$ is not homotopy equivalent to S^1.