INTRODUCTION TO TOPOLOGICAL K-THEORY
EXERCISE SESSION 10

June 14, 2016

Problem 1. Show that the induced map $f^*\colon \tilde{K}_{\mathbf{F}}^{\leq 0}(Y) \to \tilde{K}_{\mathbf{F}}^{\leq 0}(X)$ only depends on the pointed homotopy class of $f\colon X \to Y$.

Problem 2. Let X be a pointed compact Hausdorff space with basepoint x_0. Show that the reduced theory $\tilde{K}_{\mathbf{F}}^{\leq 0}(X)$ can be recovered from the unreduced theory $K_{\mathbf{F}}^{\leq 0}(X)$ as the kernel of the map

$$K_{\mathbf{F}}^{\leq 0}(X) \longrightarrow K_{\mathbf{F}}^{\leq 0}(x_0)$$

induced by the inclusion of the basepoint into X.

Problem 3. Let X be a pointed compact Hausdorff space. Show that

$$K_{\mathbf{F}}^{\leq 0}(X) \approx \tilde{K}_{\mathbf{F}}^{\leq 0}(X) \oplus K_{\mathbf{F}}^{\leq 0}(\mathrm{pt}).$$

Problem 4. Prove Lemma XVI.8.

Problem 5. Use the unreduced external product to show that $K_{\mathbf{F}}^{\leq 0}(\mathrm{pt})$ is a graded ring (with unit). Give the definition of a graded module over a graded ring, and show that for any compact Hausdorff space X, the K-theory $K_{\mathbf{F}}^{\leq 0}(X)$ is a graded module over $K_{\mathbf{F}}^{\leq 0}(\mathrm{pt})$.